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Lower Bounds for Linear Forms in Values of Polylogarithms

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Abstract—

Using Hermite–Padé approximations of the second kind, we prove a lower estimate for the absolute value of a linear form with integer coefficients in values of polylogarithmic functions at a rational point. The estimate takes account of all coefficients of the linear form.
Key words: *linear independence, polylogarithm, Hermite–Padé approximation.*

Consider the polylogarithmic functions

$$L_k(z)=\sum_{\nu=1}^{\infty}\frac{z^{\nu}}{\nu^k},\qquad k\ge 1,\quad |z|<1. \tag{1}$$

Using Hermite–Padé approximations of the second kind, a lower estimate for the absolute value of a linear form with integer coefficients in values of functions (1) at a rational point is obtained in the work. The estimate depends on all coefficients of the form.

Theorem 1. *Let $b\in\mathbb{Z}$, $a\in\mathbb{N}$, $a^{m+1}e^{m^2+m\log m}<|b|$. Then there exists a positive constant $c=c(a,b,m)>0$ such that for any integers $x_0,x_1,\dots,x_m$, not all zeros, and for $\overline{x}_k\ge\max(1,|x_k|)$, $k=1,\dots,m$, satisfying the conditions $\overline{x}_1\ge\overline{x}_2\ge\dots\ge\overline{x}_m$, the following inequality holds:*

$$\left|x_0+\sum_{k=1}^{m}x_kL_k\left(\frac{a}{b}\right)\right|>c\cdot(\overline{x}_1\cdots\overline{x}_m)^{-1}\cdot\overline{x}_1^{-\delta},$$

where

$$\delta=\frac{m^3+m^2\log m+m^2+m(m+2)\log 2+m(m+1)\log a}{\log|b|-(m+1)\log a-m^2-m\log m}. \tag{2}$$

Corollary 1. *Let $b\in\mathbb{Z}$, $a\in\mathbb{N}$, $\varepsilon>0$, and*

$$|b|^{\varepsilon/(m+\varepsilon)}>a^{m+1}e^{m^2+m\log m+4m}.$$

Then, for any non-zero collection of integers $x_0,x_1,\dots,x_m$ and numbers $\overline{x}_k\ge\max(1,|x_k|)$, $k=1,\dots,m$, satisfying the conditions $\overline{x}_1\ge\overline{x}_2\ge\dots\ge\overline{x}_m$, $\overline{x}_1\ge X_0=X_0(a,b,m)>0$, the following inequality holds:

$$\left|x_0+\sum_{k=1}^{m}x_kL_k\left(\frac{a}{b}\right)\right|>(\overline{x}_1\cdots\overline{x}_m)^{-1}\cdot\overline{x}_1^{-\varepsilon}.$$

Dirichlet's box principle shows that sharpening of the last inequality is possible at the expense of the quantity ε only.

A detailed history for the statement of the problem may be found in [1, Chap. 5, § 6]. Note also that the first estimate of a linear form in the numbers e^{α_i}, $\alpha_i \in \mathbb{Q}$, $i = 1, \dots, m$, which depends on all coefficients, has been obtained by A. Baker in [2]. Such estimates for linear forms involving values of certain concrete hypergeometric E-functions are obtained in the works [3]–[5], and involving values of an arbitrary collection of E-functions satisfying a system of linear differential equations of order 1, provided certain additional restrictions, is proved in the work [6].

Concerning values of G-functions, estimates of the above type are proved for only a few collections of functions [7]–[11]. Theorem 1 makes more precise the result in [10]. This is achieved thanks to application of a construction of Hermite–Padé approximations to the polylogarithmic functions of the second kind, and not of the first kind as in the paper [10]. Note that diagonal Hermite–Padé approximations of the second kind to the polylogarithms have been already used in the works [12]–[14] in order to deduce irrationality results for the values of functions (1) and lower bounds for corresponding linear forms with dependence on the maximum of the coefficients of the form. Functional properties of matrix Hermite–Padé approximations of the polylogarithms are studied in [15].

Let us now proceed to the proof of Theorem 1.

1. CONSTRUCTION OF APPROXIMATING FORMS

Let $n_1, \dots, n_m$ be positive integers, $n_1 \ge \dots \ge n_m$, and $N = \sum_{k=1}^m n_k$, $\mathbf{n} = (n_1, \dots, n_m)$. Define

$$R_{\mathbf{n}}(\nu) = \frac{(-1)^N \cdot N!}{n_1! \cdots n_m!} \cdot \frac{(\nu - n_1)_{n_1} (\nu - n_2)_{n_2} \cdots (\nu - n_m)_{n_m}}{\nu(\nu+1)\cdots(\nu+N)}, \tag{3}$$

where $(\nu)_0 = 1$ and $(\nu)_k = \nu(\nu+1)\cdots(\nu+k-1)$ if $k \ge 1$.

For $\mu = 1, 2, \dots, m$ consider the functions

$$E_{\mathbf{n},\mu}(z) = \frac{(-1)^{\mu-1}}{(\mu-1)!} \sum_{\nu=1}^{\infty} R_{\mathbf{n}}^{(\mu-1)}(\nu) z^{-\nu}. \tag{4}$$

Since $R_{\mathbf{n}}(\nu) = O(1/\nu)$ as $\nu \to \infty$, the series in (4) absolutely converge for $z \in \mathbb{C}$, $|z| > 1$. Denote by D_N the least common multiple of the numbers $1, 2, \dots, N$.

Lemma 1. *For $\mathbf{n} \in \mathbb{N}^m$, $\mu = 1, 2, \dots, m$ and $|z| > 1$, we have the equality*

$$E_{\mathbf{n},\mu}(z) = Q_{\mathbf{n}}(z) \cdot L_\mu\left(\frac{1}{z}\right) - P_{\mathbf{n},\mu}(z), \tag{5}$$

where

$$Q_{\mathbf{n}}(z) \in \mathbb{Z}[z], \quad D_N^\mu P_{\mathbf{n},\mu}(z) \in \mathbb{Z}[z], \qquad \deg Q_{\mathbf{n}}(z) = N, \quad \deg P_{\mathbf{n},\mu}(z) \le N-1,$$

and

$$\operatorname{ord}_{z=\infty} E_{\mathbf{n},\mu}(z) \ge n_\mu + 1. \tag{6}$$

Proof. Decomposing $R_{\mathbf{n}}(\nu)$ in the sum of partial fractions we obtain

$$R_{\mathbf{n}}(\nu) = \sum_{k=0}^{N} \frac{\omega_{\mathbf{n},k}}{\nu + k},$$

where

$$\omega_{\mathbf{n},k} = R_{\mathbf{n}}(\nu)(\nu+k)\big|_{\nu=-k} = (-1)^k \binom{N}{k} \prod_{j=1}^{m} \binom{k+n_j}{n_j} \in \mathbb{Z}. \tag{7}$$

Therefore, by (4) we see that

$$E_{\mathbf{n},\mu}(z) = \sum_{k=0}^{N}\sum_{\nu=1}^{\infty} \frac{\omega_{\mathbf{n},k}}{(\nu+k)^\mu} z^{-\nu} = \sum_{k=0}^{N} \omega_{\mathbf{n},k} z^k \cdot \sum_{\nu=1}^{\infty} \frac{z^{-(\nu+k)}}{(\nu+k)^\mu} = Q_{\mathbf{n}}(z) \cdot L_\mu\left(\frac{1}{z}\right) - P_{\mathbf{n},\mu}(z),$$

where

$$Q_{\mathbf{n}}(z) = \sum_{k=0}^{N} \omega_{\mathbf{n},k} z^k \in \mathbb{Z}[z], \quad P_{\mathbf{n},\mu}(z) = \sum_{k=0}^{N}\sum_{l=1}^{k} \frac{\omega_{\mathbf{n},k}}{l^\mu} z^{k-l} \in \mathbb{Q}[z], \tag{8}$$
$$\deg Q_{\mathbf{n}}(z) = N, \quad \deg P_{\mathbf{n},\mu}(z) \le N-1.$$

In accordance with inclusions (7) and

$$D_N^\mu \cdot \sum_{l=1}^{k} \frac{1}{l^\mu} \in \mathbb{Z}, \qquad k = 1, 2, \dots, N,$$

we conclude that $D_N^\mu \cdot P_{\mathbf{n},\mu} \in \mathbb{Z}[z]$.

Inequality (6) is implied by (4) and (3), and the lemma follows. □

Lemma 2. *If $|z| > 1$, the following inequality is valid:*

$$|Q_{\mathbf{n}}(z)| < (N+1) \cdot 2^{(m+2)N} |z|^N.$$

Proof. By (7) and (8), we have

$$Q_{\mathbf{n}}(z) = \sum_{k=0}^{N} (-1)^k \binom{N}{k} \binom{k+n_1}{n_1} \cdots \binom{k+n_m}{n_m} z^k, \tag{9}$$

hence the required estimate follows from the inequality $\binom{n}{k} \le 2^n$ valid for all $k = 0, 1, \dots, n$, $n \in \mathbb{N}$. □

Lemma 3. *For $|z| > 1$ and all $\mu = 1, 2, \dots, m$, the following inequality holds:*

$$|E_{\mathbf{n},\mu}(z)| \le \frac{2^\mu \cdot m^N \cdot |z|^{-n_\mu}}{|z|-1}.$$

Proof. From (4) and (6) we deduce that

$$|E_{\mathbf{n},\mu}(z)| \le \frac{1}{(\mu-1)!} \sum_{\nu=n_\mu+1}^{\infty} |R_{\mathbf{n}}^{(\mu-1)}(\nu)| \cdot |z|^{-\nu}, \tag{10}$$

where

$$R_{\mathbf{n}}^{(\mu-1)}(\nu) = \frac{(\mu-1)!}{2\pi i} \oint_{|\xi-\nu|=1/2} \frac{R_{\mathbf{n}}(\xi)}{(\xi-\nu)^\mu}\, d\xi. \tag{11}$$

For all integers k and complex numbers $\xi = \nu + e^{i\varphi}/2$, $0 \le \varphi \le 2\pi$, $\nu \in \mathbb{N}$, we see that $|\xi + k|^2 = (\nu + k)^2 + (\nu + k)\cos\varphi + 1/4$; this yields

$$|\nu + k| - \frac{1}{2} \le |\xi + k| \le |\nu + k| + \frac{1}{2}. \tag{12}$$

Therefore, for $\nu > 1$ and $n \in \mathbb{N}$ we have

$$|(\xi - n)_n| \le |\xi - 1| \cdot (|\xi - 1| + 1) \cdots (|\xi - 1| + n - 1) \le \left(\nu - \frac{1}{2}\right)_n. \tag{13}$$

Furthermore, with the help of (3), (12) and (13) we deduce the estimate

$$|R_{\mathbf{n}}(\xi)| \le \frac{N!}{n_1! \cdots n_m!} \cdot \frac{(\nu - 1/2)_{n_1} (\nu - 1/2)_{n_2} \cdots (\nu - 1/2)_{n_m}}{(\nu - 1/2)(\nu + 1/2) \cdots (\nu + N - 1/2)}$$

on the contour of integration. Write the product in the denominator of the latter fraction as follows:

$$\left(\nu - \frac{1}{2}\right)\left(\nu + \frac{1}{2}\right) \cdots \left(\nu + N - \frac{3}{2}\right) = \left(\nu - \frac{1}{2}\right)_{n_1} \cdots \left(\nu - \frac{1}{2} + n_1 + \cdots + n_{m-1}\right)_{n_m}.$$

Then

$$|R_{\mathbf{n}}(\xi)| \le \frac{N!}{n_1! \cdots n_m!} \cdot \frac{1}{\nu + N - 1/2} \le \frac{m^N}{\nu + N - 1/2}.$$

Finally, by (11) we obtain

$$|R_{\mathbf{n}}^{(\mu-1)}(\nu)| \le \frac{(\mu - 1)! \cdot 2^{\mu-1} \cdot m^N}{\nu + N - 1/2} \le \frac{(\mu - 1)! \cdot 2^{\mu} \cdot m^N}{\nu + N},$$

and the statement of the lemma now follows from (10). □

Define the collection of multi-indices

$$\mathbf{n}_j = (n_1 + 1, \dots, n_j + 1, n_{j+1}, \dots, n_m), \qquad j = 0, 1, \dots, m.$$

Consider the determinant

$$\Delta_{\mathbf{n}}(z) = \begin{vmatrix} Q_{\mathbf{n}_0}(z) & P_{\mathbf{n}_0,1}(z) & \dots & P_{\mathbf{n}_0,m}(z) \\ Q_{\mathbf{n}_1}(z) & P_{\mathbf{n}_1,1}(z) & \dots & P_{\mathbf{n}_1,m}(z) \\ \vdots & \vdots & \ddots & \vdots \\ Q_{\mathbf{n}_m}(z) & P_{\mathbf{n}_m,1}(z) & \dots & P_{\mathbf{n}_m,m}(z) \end{vmatrix}. \tag{14}$$

Lemma 4. *We have*

$$\Delta_{\mathbf{n}}(z) \equiv \mathrm{const} \ne 0.$$

Proof. Lines and columns of determinant (14) will be indexed from 0 to m. First of all, note that the determinant is a polynomial. Further, for each $j = 1, \dots, m$, subtract the 0th column multiplied by $L_j(1/z)$ from the jth column of the determinant $\Delta_{\mathbf{n}}(z)$. By (5) we obtain

$$\Delta_{\mathbf{n}}(z) = (-1)^m \begin{vmatrix} Q_{\mathbf{n}_0}(z) & E_{\mathbf{n}_0,1}(z) & \dots & E_{\mathbf{n}_0,m}(z) \\ Q_{\mathbf{n}_1}(z) & E_{\mathbf{n}_1,1}(z) & \dots & E_{\mathbf{n}_1,m}(z) \\ \vdots & \vdots & \ddots & \vdots \\ Q_{\mathbf{n}_m}(z) & E_{\mathbf{n}_m,1}(z) & \dots & E_{\mathbf{n}_m,m}(z) \end{vmatrix}. \tag{15}$$

Let $\Delta_{\mathbf{n},j}(z)$, $0 \le j \le m$, be the algebraic complement of the element $Q_{\mathbf{n}_j}(z)$ in determinant (15):

$$\Delta_{\mathbf{n},j}(z) = (-1)^j \begin{vmatrix} E_{\mathbf{n}_0,1}(z) & \dots & E_{\mathbf{n}_0,m}(z) \\ \dots & \dots & \dots \\ E_{\mathbf{n}_{j-1},1}(z) & \dots & E_{\mathbf{n}_{j-1},m}(z) \\ E_{\mathbf{n}_{j+1},1}(z) & \dots & E_{\mathbf{n}_{j+1},m}(z) \\ \dots & \dots & \dots \\ E_{\mathbf{n}_m,1}(z) & \dots & E_{\mathbf{n}_m,m}(z) \end{vmatrix}.$$

Then

$$\Delta_{\mathbf{n}}(z) = (-1)^m \sum_{j=0}^{m} Q_{\mathbf{n}_j}(z)\Delta_{\mathbf{n},j}(z). \tag{16}$$

By Lemma 1 we see that

$$\deg Q_{\mathbf{n}_j}(z) = N + j, \quad j = 0, 1, \dots, m, \qquad \Delta_{\mathbf{n},0}(z) = O\biggl(\frac{1}{z^{N+m+1}}\biggr), \quad z \to \infty, \tag{17}$$

$$\Delta_{\mathbf{n},j}(z) = O\biggl(\frac{1}{z^{N+m}}\biggr), \qquad z \to \infty, \quad j = 1, \dots, m. \tag{18}$$

It follows from (16)–(18) that $\Delta_{\mathbf{n}}(z) = O(1)$ as $z \to \infty$, in other words, $\Delta_{\mathbf{n}}(z)$ is a polynomial of degree 0, hence $\Delta_{\mathbf{n}}(z) \equiv \Delta_{\mathbf{n}}(0)$. Thus, using (16), (9), (4), and (3) we obtain

$$\Delta_{\mathbf{n}}(z) \equiv \Delta_{\mathbf{n}}(0) = \pm \prod_{k=1}^{m} \binom{N + m + n_k + 1}{n_k + 1} \cdot \prod_{j=0}^{m-1} \frac{1}{j!} R_{\mathbf{n}_j}^{(j)}(n_{j+1} + 1) \ne 0,$$

and the lemma is proved. $\square$

2. PROOF OF THEOREM 1

Let $z = b/a$, where the numbers $a \in \mathbb{N}$ and $b \in \mathbb{Z}$ satisfy the condition

$$|b| > a^{m+1} \cdot e^{m^2 + m\log m}. \tag{19}$$

In particular, $|z| > 1$. Consider a form

$$\ell = x_0 + x_1 L_1\left(\frac{a}{b}\right) + \dots + x_m L_m\left(\frac{a}{b}\right) \tag{20}$$

with arbitrary integer coefficients, not all zeros, and assume that the numbers $\overline{x}_k \ge \max(1, |x_k|)$, $k = 1, \dots, m$, satisfy $\overline{x}_1 \ge \dots \ge \overline{x}_m$. Take the positive integer n_1 as the least number satisfying the condition

$$\overline{x}_1 \cdot \left(e^{m^2 + m\log m} \cdot \frac{a^{m+1}}{|b|}\right)^{n_1} < \frac{1}{2m};$$

by (19) for any $\overline{x}_1$ sufficiently large, the number n_1 always exists. For $1 < k \le m$, take n_k as the least possible positive integer satisfying the condition

$$\overline{x}_k \cdot (e^{m^2 + m\log m} \cdot a^m)^{n_1} \cdot \left(\frac{a}{|b|}\right)^{n_k} < \frac{1}{2m}.$$

It can be easily seen that the above choice of the numbers n_k ensures $n_1 \ge n_2 \ge \dots \ge n_m$. Moreover, when a and b are fixed, the three conditions $\overline{x}_1 \to \infty$, $n_1 \to \infty$ and $N \to \infty$ are equivalent. Further, in accordance with Lemma 4, there exists an index $j=0,1,\dots,m$ depending on $\mathbf{n}$ such that

$$x_0 Q_{\mathbf{n}_j}\left(\frac{b}{a}\right)+\sum_{k=1}^{m} x_k P_{\mathbf{n}_j,k}\left(\frac{b}{a}\right)\neq 0. \tag{21}$$

Multiply both sides of equality (20) by $a^{N+m}D_{N+m}^{m}Q_{\mathbf{n}_j}(b/a)$ and write the result in the form

$$\begin{aligned} a^{N+m}D_{N+m}^{m}Q_{\mathbf{n}_j}\left(\frac{b}{a}\right)\ell &= a^{N+m}D_{N+m}^{m}\left(x_0 Q_{\mathbf{n}_j}\left(\frac{b}{a}\right)+\sum_{k=1}^{m} x_k P_{\mathbf{n}_j,k}\left(\frac{b}{a}\right)\right) \\ &\quad + a^{N+m}D_{N+m}^{m}\sum_{k=1}^{m} x_k E_{\mathbf{n}_j,k}\left(\frac{b}{a}\right). \end{aligned} \tag{22}$$

By Lemma 1 the first term in the right-hand side of (22) is an integer, which is not zero due to (21). Therefore, the absolute value of the integer is at least 1. This implies the inequality

$$a^{N+m}D_{N+m}^{m}\left|Q_{\mathbf{n}_j}\left(\frac{b}{a}\right)\right|\cdot|\ell|\geq 1-a^{N+m}D_{N+m}^{m}\left|\sum_{k=1}^{m} x_k E_{\mathbf{n}_j,k}\left(\frac{b}{a}\right)\right|.$$

By Lemmas 2 and 3, for $|z|>1$ and all N sufficiently large, the following estimates are valid:

$$\begin{gathered} a^{N+m}D_{N+m}^{m}\leq A\cdot e^{mN}a^{N}, \\ \left|Q_{\mathbf{n}_j}\left(\frac{b}{a}\right)\right|\leq B\cdot 2^{(m+2)N}\left(\frac{|b|}{a}\right)^{N}, \qquad \left|E_{\mathbf{n}_j,k}\left(\frac{b}{a}\right)\right|\leq C\cdot m^{N}\left(\frac{|b|}{a}\right)^{-n_k}, \end{gathered} \tag{23}$$

where $A=e^{o(N)}$, $B=e^{o(N)}$, and $C=e^{o(N)}$ as $N\to\infty$.

Applying inequalities (23) we see that, for all N sufficiently large,

$$e^{mN}a^{N}\cdot 2^{(m+2)N}\left(\frac{|b|}{a}\right)^{N}|\ell|\geq 1-e^{mN}a^{N}\sum_{k=1}^{m}\overline{x}_k m^{N}\left(\frac{a}{|b|}\right)^{n_k}. \tag{24}$$

Recalling our choice of the numbers $\overline{x}_k$, $k=1,\dots,m$, we obtain that, for all n_1 greater than some number depending on a and b, the right-hand side of inequality (24) is greater then $1/2$, hence

$$\begin{aligned} |\ell| &> \frac{1}{2}e^{-(m+(m+2)\log 2)N}|b|^{-N} > c(\overline{x}_1\cdots\overline{x}_m)^{-1}e^{-(m^3+m^2\log m)n_1}a^{-N-m^2n_1}e^{-(m+(m+2)\log 2)N} \\ &> c_*(\overline{x}_1\cdots\overline{x}_m)^{-1}\left(a^{m^2+m}e^{m^3+m^2\log m+m^2+m(m+2)\log 2}\right)^{-n_1} > c_{**}(\overline{x}_1\cdots\overline{x}_m)^{-1}\cdot\overline{x}_1^{-\delta}, \end{aligned}$$

where $c_{**}=c_{**}(a,b,m)$ and δ is defined in (2). The proof of the theorem is complete. □

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REFERENCES

1. N. I. Fel′dman and Yu. V. Nesterenko, *Transcendental numbers*, Number theory, IV, Encyclopaedia Math. Sci. **44**, Springer, Berlin, 1998.

2. A. Baker, On some Diophantine inequalities involving the exponential function, *Canad. J. Math.* **17** (1965), 616–626.
3. N. I. Fel′dman, Estimates from below for certain linear forms, *Vestnik Moskov. Univ. Ser. I Mat. Meh.* **22** (1967), no. 2, 63–72. (Russian)
4. K. Väänänen, On lower estimates for linear forms involving certain transcendental numbers, *Bull. Austral. Math. Soc.* **14** (1976), 161–179.
5. K. Väänänen, On linear forms of the values of one class of E-functions, *Acta Univ. Ouluen Ser. A* (1976), no. 41, 1–19.
6. V. V. Zudilin [W. Zudilin], Lower bounds for polynomials in the values of certain entire functions, *Mat. Sb.* **187** (1996), no. 12, 57–86; English translation in: *Sb. Math.* **187** (1996), no. 12, 1791–1818.
7. N. I. Fel′dman, A certain linear form, *Acta Arith.* **21** (1972), 347–355. (Russian)
8. V. N. Sorokin, Irrationality of values of hypergeometric functions, *Mat. Sb.* (*N.S.*) **127** (**169**) (1985), no. 2, 245–258; English translation in: *Math. USSR-Sb.* **55** (1986), no. 1, 243–257.
9. A. I. Galochkin, Lower bounds of linear forms in the values of certain G-functions, *Mat. Zametki* **18** (1975), no. 4, 541–552; English translation in: *Math. Notes* **18** (1975), no. 4, 911–917.
10. T. Hessami Pilehrood [T. G. Khessami Pilerud], A lower bound for a linear form, *Mat. Zametki* **66** (1999), no. 4, 617–623; English translation in: *Math. Notes* **66** (1999), no. 3–4, 507–512.
11. T. Hessami Pilehrood [T. G. Khessami Pilerud], Lower bounds for linear forms in values of some hypergeometric functions, *Mat. Zametki* **67** (2000), no. 3, 441–451; English translation in: *Math. Notes* **67** (2000), no. 3–4, 372–381.
12. G. V. Chudnovsky, Padé approximations to the generalized hypergeometric functions. I, *J. Math. Pures Appl.* **58** (1979), 445–476.
13. L. A. Gutnik, Linear independence over $\mathbb{Q}$ of dilogarithms at rational points, *Uspekhi Mat. Nauk* **37** (1982), no. 5, 179–180. (Russian)
14. M. Hata, On the linear independence of the values of polylogarithmic functions, *J. Math. Pures Appl.* **69** (1990), no. 2, 133–173.
15. V. N. Sorokin, Hermite–Padé approximants of polylogarithms, *Izv. Vyssh. Uchebn. Zaved. Mat.* (1994), no. 5, 49–59. (Russian)

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